Coupled-Channels Scattering Solutions using the R-matrix Method

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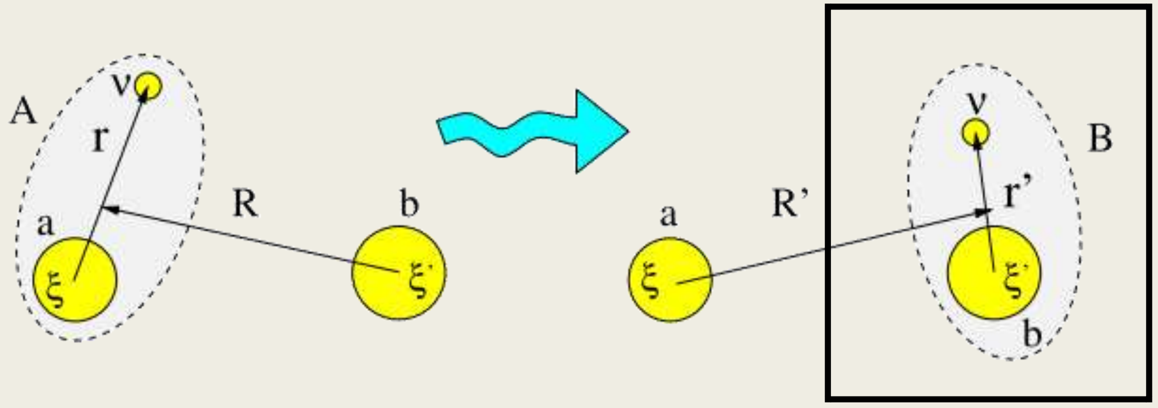
Motivation

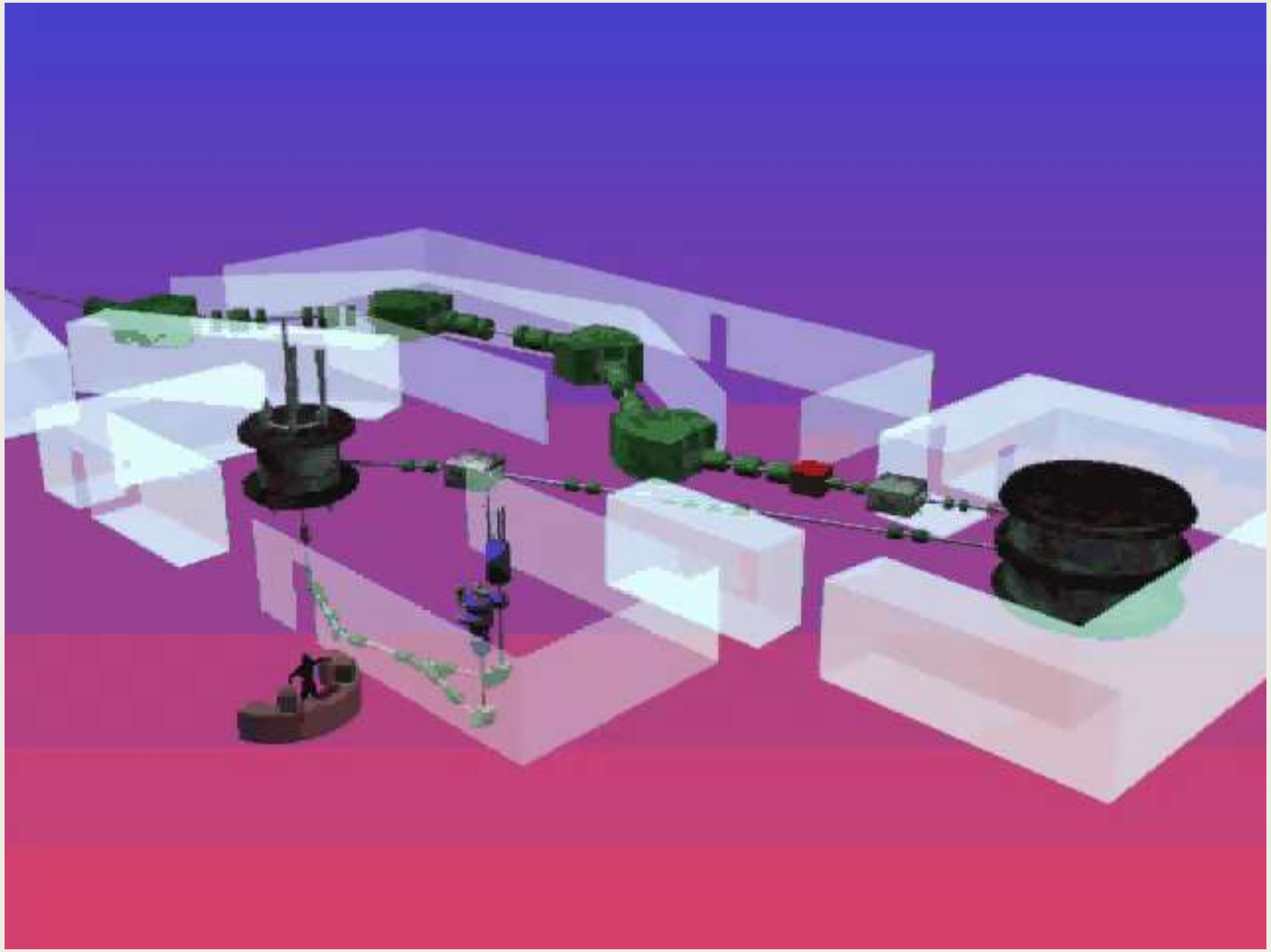
- Goal is to calculate wavefunctions for coupled channels scattering reactions with nonlocal interactions

$$\left[-\frac{\hbar^2}{2\mu_c}\left(\frac{d^2}{dr^2}-\frac{l_c(l_c+1)}{r^2}\right)+V_c(r)+E_c-E\right]u_{c(c_0)}(r)+\sum_{c'}\int_0^\infty W_{cc'}(r,r')u_{c'(c_0)}(r')dr'=0$$

- R-matrix method offers an efficient framework for calculating such solutions.



Context



R-matrix Method

- Radial space the wavefunction will be solved for is divided at a channel radius
- Inside the channel radius, the wavefunction is calculated over a finite number of basis functions:

$$C_{ci,c'i'} = < \varphi_i | T_c + \mathcal{L}_c + E_c - E | \varphi_{i'} > \delta_{cc'} + < \varphi_i | V_{cc'} | \varphi_{i'} >$$

- In the exterior region, the asymptotic behavior of the wavefunction can be modeled using the collision matrix of the system and Coulomb functions[1]:

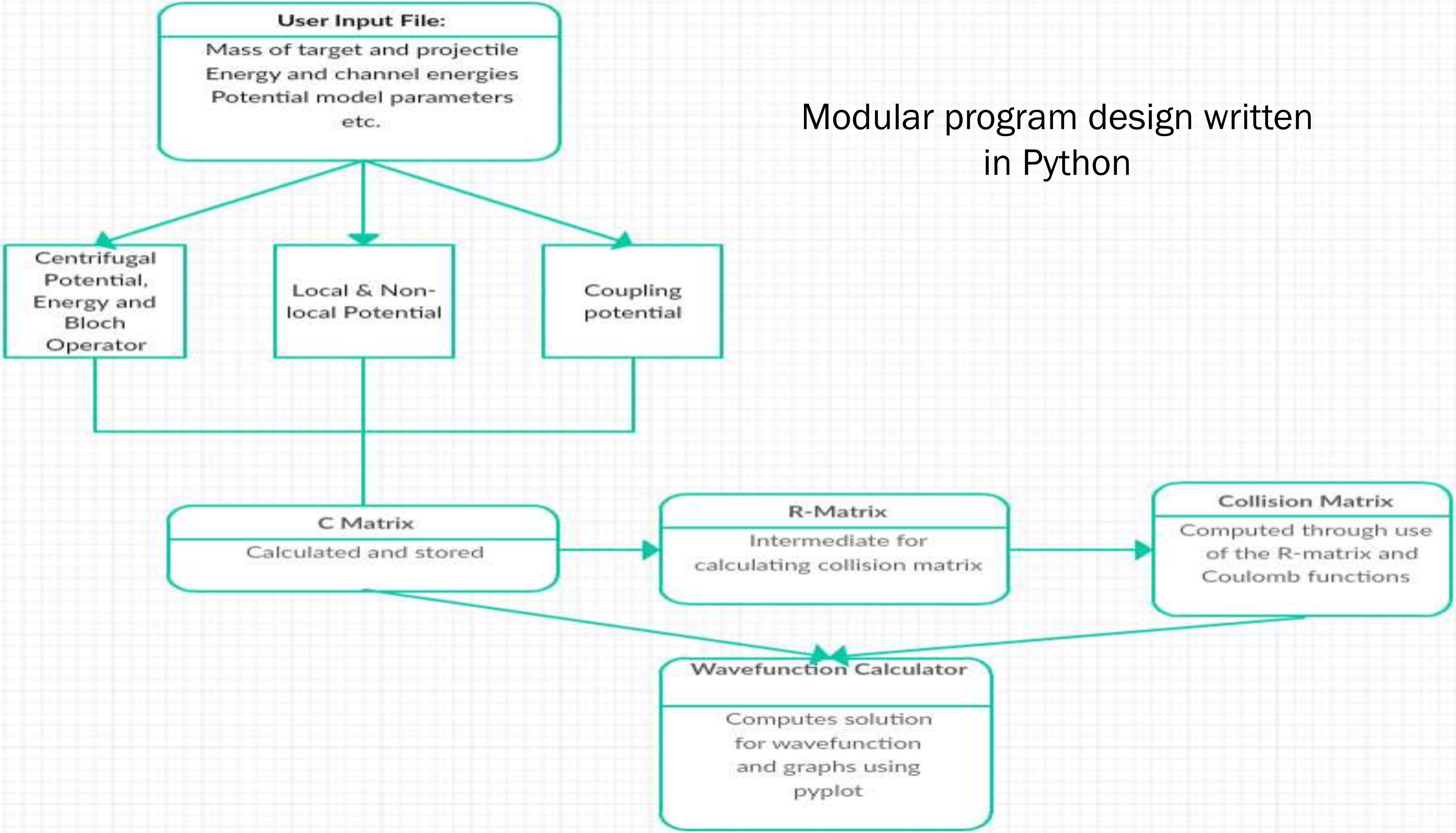
$$u_{c,ext} = v_c^{-1/2} (I_c(k_c r)\delta_{cc_0} - U_{cc_0} O_c(k_c r))$$

[1]: Descouvemont, P., & Baye, D. (2010). The R -matrix theory. *Reports on Progress in Physics*, *73*(3), 036301.

Inputs

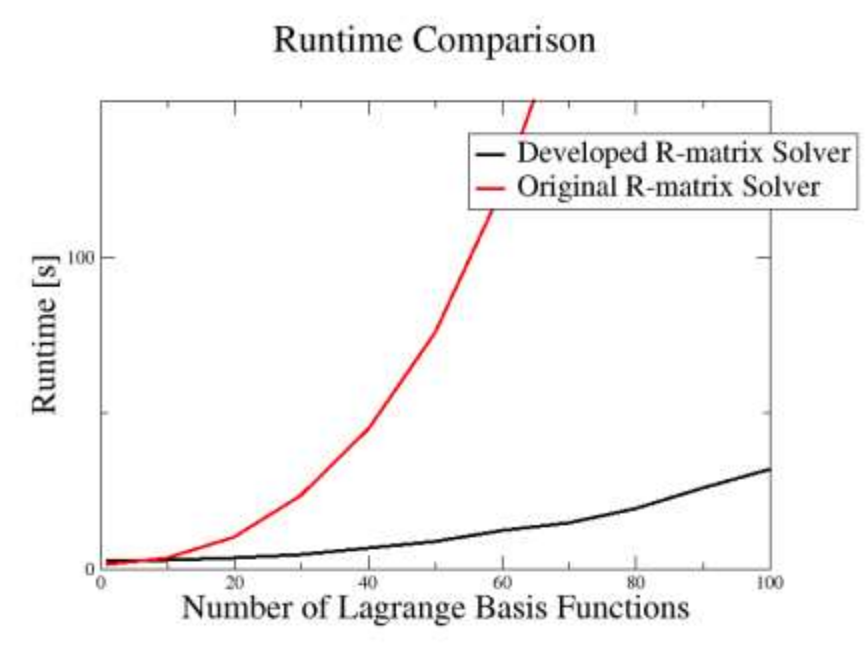
- Information about the system:
  - *Energy of the incoming nucleon (E) and excitation energies (E_c)*
  - *Angular momentum values (l)*
  - *Reduced mass of the system (μ)*
- Parameterization of potentials (V(r))
  - *Both local and nonlocal*
  - *Model interaction between two particles in a channel and between the two coupled channels*
  - *Usual potential shape is of the Woods-Saxon form:* $V_{WS}(r) = \frac{-V_0}{1+e^{\frac{r-R}{a}}}$
  - *Coupling potential model:* $V_{coupling} = \beta_{coupling} \frac{d}{dr} \frac{V_c}{1 - e^{\frac{r-R_c}{a_c}}}$

Modular program design written in Python



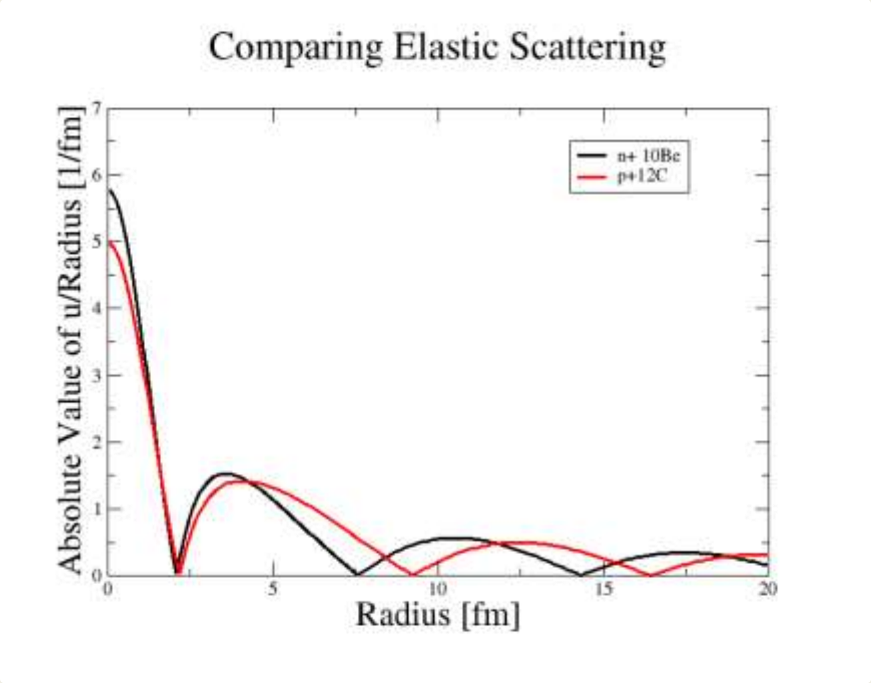
Program Development

- Began with single channel R-matrix solver
- Generalized to calculate two coupled channel problems
- Optimized by performing as few matrix calculations as possible
- Large matrices calculated once and passed as parameters where needed

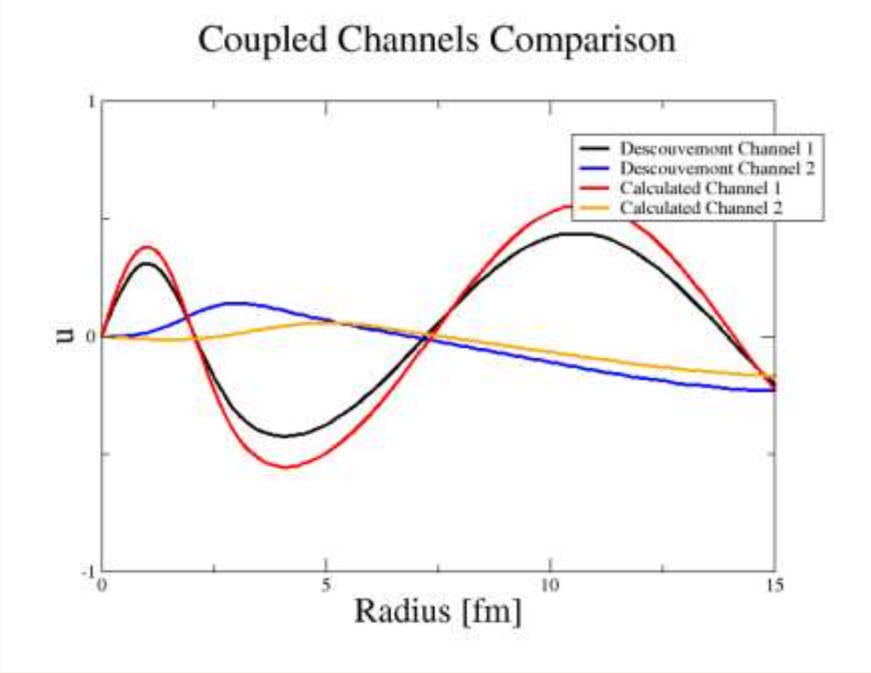


Preliminary Results

Single-channel wavefunctions for elastic scattering: n+^{10}Be E=5 MeV , p+^{12}C both at 5 MeV.



Comparing with Fortran package[2]: n+^{10}Be at 5 MeV with E_c=3.368 MeV (local coupled-channel case).



[2]:Descouvemont, P. (2016). An R-matrix package for coupled-channel problems in nuclear physics. *Computer Physics Communications*, *200*, 199–219.

Future Improvements

After more testing ensures the accuracy of the developed coupled channels solution, there are many opportunities for future work:

- Generalization for arbitrary number of channels
- Include parallel programming techniques.
- Expand the solver to three-body problems.

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