Introduction

- Anomalous, power-law based phenomena are ubiquitous
- Anomalous biomechanics, Figs. 1 & 2
- Eddies in turbulent flows, Fig. 3
- Anomalous diffusion in transient media

Methods

- Three methods were benchmarked with Eqn. 2, a standard FODE
- Petrov-Galerkin Spectral Method (PGSM) [2]
- Finite Difference Method (FDM) [2]
- Finite Element Method (FEM) [3]

\[ \frac{RLD^\nu}{0} u(t) = f(t), \ u(0) = 0 \] (2)

- Tested with method of fabricated solution for CPU time and error,
- \( E_{L_2} = \frac{|u^{ex}(t) - u^{pp}(t)|}{|u^{ex}(t)|} \)
- Schemes are applied to anomalous Maxwell material stress-strain behavior via Eqn. 3 [1]

\[ \nu RL_{0} D^\nu \epsilon(t) = \sigma(t) + G RL_{0} D^{\nu-\mu} \sigma(t) \] (3)

where \( \epsilon(t) \) and \( \sigma(t) \) are the strain and stress of the material

Results

- Fig. 4 shows a sample fit of the test benchmark function \( RL_{0} D^0.2 \mu = 1 \) \( \Gamma(6.1) \) \( \Gamma(5.9) \) using PGSM

\[ RL_{0} D^0.2 \mu \Gamma(6.1) \Gamma(5.9) \] (1)

Kernel captures power law

Integration captures history

- Low CPU time and high accuracy are desired
- Benchmark criterion required to quantify computational efficacy
- Thus, we develop an assessment framework to evaluate numerical schemes
- R platform works well with large sets of data

Discussion

- Framework records evaluation criterion for schemes, listed in Fig. 5 and Table 1
- Table 2 lists insight into beneficial algorithm mechanics

Figure 5. Rate of convergence for \( RL_{0} D^0.2 \mu = 1 \) \( \Gamma(6.1) \) \( \Gamma(5.9) \)

PGSM demonstrates absolute convergence at 6 terms.

Table 1 lists CPU times required to reach \( O(10^{-5}) \) error for each method

<table>
<thead>
<tr>
<th>Method</th>
<th>( \tilde{E}<em>{L_2} ) at ( N</em>{pol} + N_el = 6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGSM</td>
<td>2e-15</td>
</tr>
<tr>
<td>FDM</td>
<td>6e-1</td>
</tr>
<tr>
<td>FEM</td>
<td>2e-1</td>
</tr>
</tbody>
</table>

Future Work

- Extension to fractional diffusion equation numerical schemes
- Implement data fitting performance metrics for large data sets

Figure 6. Simulations of strain, \( \epsilon(t) \), versus test stress function, \( \sigma(t) = 0.9 |v^{\nu-\mu} - 1^{\nu-\mu}| \) with \( v = 0.9, \mu = 0.7, \) and \( v = 1 \) and various \( \nu \) and \( G \)

Table 2. Key features for various methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Key Feature</th>
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<tbody>
<tr>
<td>PGSM</td>
<td>Diagonal linear system</td>
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<tr>
<td>FDM</td>
<td>Generalizable form</td>
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<tr>
<td>FEM</td>
<td>Adaptive to irregularities</td>
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Table 3. Pros and cons for various methods

<table>
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<th>Pros</th>
<th>Cons</th>
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</table>
| PGSM       | - Hard to implement
|            | smooth behavior
|            | - Limited to smooth
|            | functions
| FDM        | - Large history
|            | iteration
| FEM        | - High accuracy
|            | unfeasible

References


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