Coupled-Channels Scattering Solutions using the R-matrix Method

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Motivation

- Goal is to calculate wavefunctions for coupled channels scattering reactions with nonlocal interactions

\[
\left[-\frac{\hbar^2}{2\mu_c} \left(\frac{d^2}{dr^2} - \frac{l_c(l_c+1)}{r^2}\right) + V_c(r) + E_c - E\right] u_{c(c_0)}(r) + \sum_{c'} \int_0^\infty W_{cc'}(r, r') u_{c'(c_0)}(r') dr' = 0
\]

- R-matrix method offers an efficient framework for calculating such solutions.
R-matrix Method

- Radial space the wavefunction will be solved for is divided at a channel radius.
- Inside the channel radius, the wavefunction is calculated over a finite number of basis functions:

  \[ C_{ci,c'i'} = \langle \phi_i | T_c + L_c + E_c - E | \phi_i' \rangle \delta_{cc'} + \langle \phi_i | V_{cc'} | \phi_i' \rangle \]

- In the exterior region, the asymptotic behavior of the wavefunction can be modeled using the collision matrix of the system and Coulomb functions\[^1\]:

  \[ u_{c,ext} = v_c^{-1/2}(I_c(k_cr)\delta_{cc0} - U_{cc0}O_c(k_cr)) \]

Inputs

- Information about the system:
  - Energy of the incoming nucleon \((E)\) and excitation energies \((E_c)\)
  - Angular momentum values \((l)\)
  - Reduced mass of the system \((\mu)\)

- Parameterization of potentials \((V(r))\)
  - Both local and nonlocal
  - Model interaction between two particles in a channel and between the two coupled channels
  - Usual potential shape is of the Woods-Saxon form:
    \[ V_{WS}(r) = \frac{-V_0}{1 + e^{\frac{r-R}{a}}} \]
  - Coupling potential model:
    \[ V_{coupling} = \beta_{coupling} \frac{d}{dr} \frac{V_c}{1 - e^{\frac{r-R_c}{a_c}}} \]
Modular program design written in Python
Program Development

- Began with single channel R-matrix solver
- Generalized to calculate two coupled channel problems
- Optimized by performing as few matrix calculations as possible
- Large matrices calculated once and passed as parameters where needed
Preliminary Results

Single-channel wavefunctions for elastic scattering: $n^+{^{10}}{Be}$ $E=5$ MeV, $p^+{^{12}}{C}$ both at 5 MeV.

Comparing with Fortran package[2]: $n^+{^{10}}{Be}$ at 5 MeV with $E_c=3.368$ MeV (local coupled-channel case).

Future Improvements

After more testing ensures the accuracy of the developed coupled channels solution, there are many opportunities for future work:

• Generalization for arbitrary number of channels
• Include parallel programming techniques.
• Expand the solver to three-body problems.
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